TIME-FREQUENCY REPRESENTATIONS OF SEISMIC SIGNALS

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Introduction

A Fourier Transform of a building record contains information regarding frequency content, but it can not resolve the exact onset of changes in natural frequency – all temporal resolution is contained in the phase of the transform. The spectrogram is better able to resolve temporal evolution of frequency content, but has a trade-off in time resolution versus frequency resolution in accordance with the uncertainty principle. To overcome this restriction, several classes of Time-Frequency representations have been proposed, including wavelet methods and quadratic time-frequency distributions such as the Wigner-Ville distribution.

Time-Frequency Representations

One method of investigating a changing signal would be to take a Fourier Transform of the first half of the signal and compare the result with the second half of the signal – or split the signal into four sections, eight, etcetera. By expanding this idea, you arrive at the spectrogram (short-time fourier transform). Effectively, by windowing the signal and taking a Fourier transform of only a slice of the signal at a time, you can identify changes in the frequency content from one slice to the next. Window length and overlap can be altered depending on the type of signal This assumes a certain amount of stationarity over the length of the slice (the same assumption that goes into the Fourier Transform, where the signal is assumed to be stationary and periodic). The spectrogram, however, cannot create a point-for point, instantaneous energy estimation – frequency resolution is inversely proportional to temporal resolution. (A wider window gives better frequency resolution, but smears information over the length of the window. Inversely, a shorter window gives better temporal resolution, at the cost of resolution in the frequency domain.)

Another representation is the Wavelet transformation: this is similar to the Fourier Transform, but instead of decomposing signals into a basis of sine waves, wavelet transforms decompose the signal into a wavelet basis (in this study, the wavelet basis used is the Morlet wavelet, a gaussian tapered cosine wave). This allows for a much finer fit to an arbitrary signal, but the results of this method do not explicitly describe the signal in terms of frequency, and the conversion from scale to frequency is only an approximation.

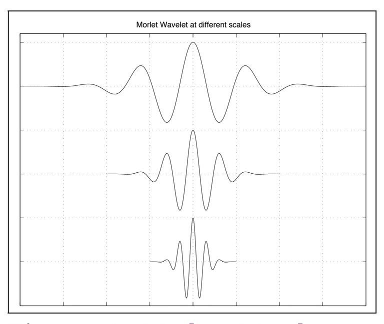

Figure 1: Morlet Wavelets at three Different Scales – gaussian tapered cosine.

The Wigner-Ville distribution, and related refinements, are a promising method for extracting instantaneous frequency estimates from a signal. As a class, these techniques provide superior temporal and frequency resolution in the time-frequency plane, which has important implications for system identification, damage detection, and structural health monitoring.

Wigner-Ville Distribution

For a signal, $s(t)$, with analytic associate $x(t)$, the Wigner-Ville Distribution, $WVD_x(t,\omega)$ is defined as:

$$WVD_x(t,\omega) = \int_{-\infty}^{+\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\mathrm{e}^{-i\omega\tau}\,d\tau$$

This distribution was first introduced by E. Wigner in the context of quantum mechanics [4], and later independently developed by J. Ville who applied the same transformation to signal processing and spectral analysis [3].

- The WVD is similar to the Fourier Transform, though, instead of transforming the original signal, the kernel of the WVD contains a type of autocorrelation term (in this case, the phase lag of the ambiguity function, or "...properly symmetrized covariance function..." [2]).
- The analytic associate $x(t)$ of a signal $s(t)$ is here defined such that $x(t) \equiv s(t) + iH[s(t)]$, where $H[s(t)]$ is the Hilbert Transform of the signal $s(t)$. (The Hilbert Transform is sometimes referred to as a "quadrature filter" and the transformed signal as the "quadrature signal," for reasons relating to the phase shifts introduced by the transform – running the transform four times will return the signal to the original signal, as each transformation shifts real frequencies by $\pi/2$ and negative frequencies by $-\pi/2$.)
- In this study, time-frequency analysis techniques are applied to the analytic associates of real signals unless noted otherwise – in particular, $x(t)$ is generally the complex-valued analytic associate of some real-valued time signal of interest.

In addition to being an entirely real-valued function, the WVD also satisfies the marginal and total energy conditions:

$$\int_{-\infty}^{+\infty} WVD_x(t,\omega)\,d\omega = |x(t)|^2$$

$$\int_{-\infty}^{+\infty} WVD_x(t,\omega)\,dt = |\mathbb{X}(\omega)|^2$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} WVD_x(t,\omega)\,dt\,d\omega = \mathbb{E}_x$$

...where $\mathbb{X}(\omega)$ is the Fourier Transform of $x(t)$, and $\mathbb{E}_x$ is the total energy of signal $x(t)$. The integral of the WVD along the time axis is equal to the power spectrum of the signal (the squared Fourier Transform), while the integral along the frequency axis gives the squared envelope of the original time series – the energy condition states that the total area of the WVD, the double integral across time and frequency, is the energy contained in the original signal. These conditions have an intuitive appeal, as they imply a limitation on the extent of the signal in the time-frequency plane. In an ideal representation, a signal of short duration would have a narrow representation along the time axis, and the frequency content would be localized to the frequency of the signal. Meeting the marginal conditions is one way in which optimal time-frequency representations can be constructed, though these conditions are neither necessary nor sufficient for the construction of useful representations.

Reduced Interference Distribution

In general, a Reduced Interference Distribution (RID) refers to any distribution that reduces the expression of the cross-terms relative to the auto-terms in a quadratic time-frequency representation [1]. One such RID uses a smoothing kernel based on a Hanning window:

$$RID(t,\omega) = \int_{-\infty}^{+\infty} h(\tau)R_x(t,\tau)\mathrm{e}^{-i\omega\tau}\,d\tau$$

With the kernel $R_x(t,\tau)$, as follows (Hanning window $\equiv [1+cos(2\pi\nu/\tau)]/2$):

$$R_x(t,\tau) = \int_{-\frac{|\tau|}{2}}^{+\frac{|\tau|}{2}} \frac{g(\nu)}{|\tau|}(1+\cos\frac{2\pi\nu}{\tau})x(t+\nu+\frac{\tau}{2})x^*(t+\nu-\frac{\tau}{2})\,d\nu$$

...where $h(\tau)$ is a time smoothing window, and $g(\nu)$ is a frequency smoothing window. Depending on the implementation, the smoothing functions $h(\tau)$ and $g(\nu)$ can be adjusted to match the requirements of the data.

Comparison of Time-Frequency Representations

When discussing "support" in the mathematical sense, the marginal conditions are one way to describe an accurate Time-Frequency representation. In an ideal representation, a signal of short duration would have a narrow representation along the time axis, and the frequency content would be localized to the frequency content of the signal.

To illustrate this point, we apply these methods to a sample signal, created to expose the strengths and weaknesses of various representations. The test signal in this case is broken into 5 main pieces of 512 points each, for a total length of 5*512=2560:

- Baseline signal which consists of a low frequency sine wave, plus noise
- Baseline plus a transient high-frequency component
- Baseline signal
- Baseline signal is doubled in amplitude, plus a transient intermediate-frequency component
- Baseline signal (doubled)

Energy in the time-frequency plane should be located at intersections of expression in the time domain and frequency domain: e.g. the first transient signal should start at the 2nd segment (of five), and it should be located vertically within the frequency indicated in the power spectrum.

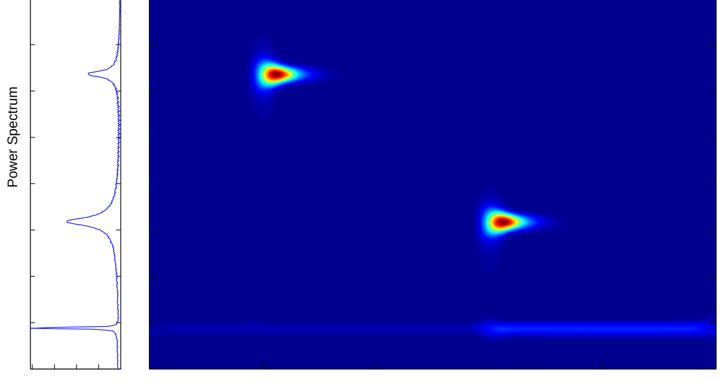

Figure 2: Spectrogram (Short-Time Fourier Transform). The spectrogram presented here has fairly good temporal and frequency support – but the unavoidable smearing of spectrogram methods results in energy appearing before the time of the transient signal onset, and the frequency content is broader than that found in the actual signal.

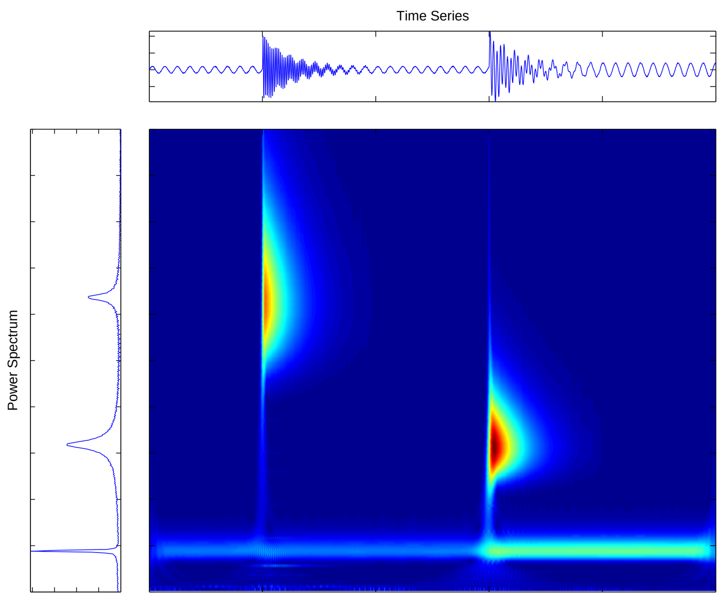

Figure 3: Continuous Wavelet Transform. The Wavelet method has better temporal support, but the frequency is poorly defined. Wavelets create a time-scale representation rather than a time-frequency representation, and there is only an approximate conversion between scale and frequency.

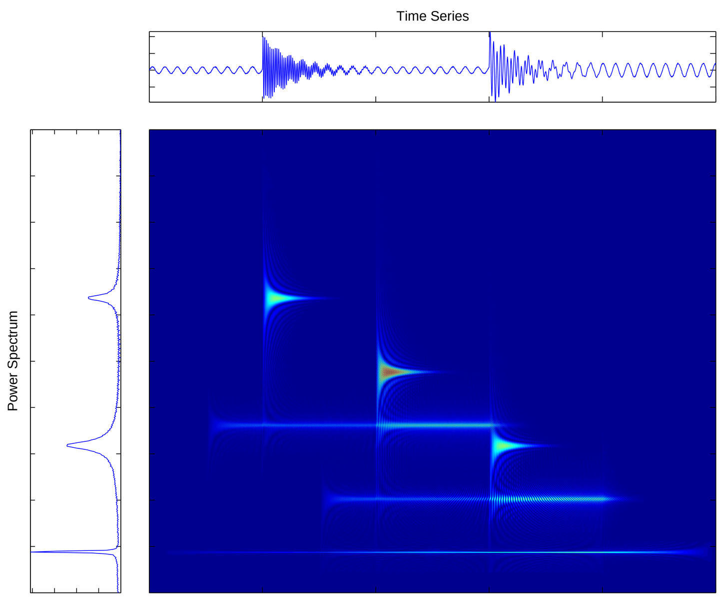

Figure 4: Wigner-Ville Distribution. This representation has excellent support from the auto terms, but cross term interference results in interference terms that do not correspond to energy in the original signal. The interference terms are highly oscillatory, and occur midway between components of the signal. In this sample signal, there are three sources of interference: Between the high-frequency transient and the baseline signal, between the low-frequency transient and the baseline, and between the two transient signals.

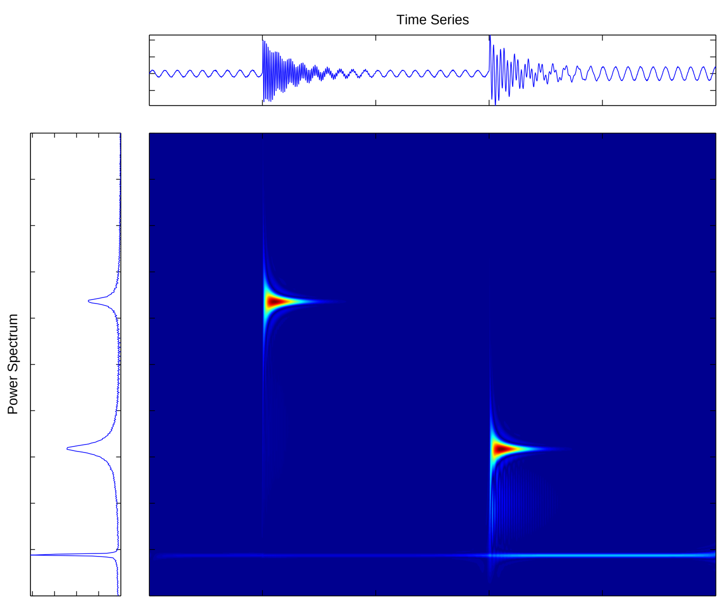

Figure 5: Reduced Interference Distribution (Wigner-Ville). It is possible to mitigate the effects of cross-term interference in a variety of ways. The RID introduces a smoothing kernel, and since the cross-term interference is highly oscillatory, smoothing is an effective way to remove these interference terms. The effects of cross-term interference cannot be completely removed, but the increased accuracy of the auto-terms (relative to the spectrogram or wavelet methods) make this representation better suited for certain types of signal processing.

Application to Seismic Records

These time-frequency methods have been successfully applied to seismic records – the changes in natural frequencies can be correlated with changes in dynamic properties, such as a decrease in stiffness from earthquake damage. Obtaining a detailed, instant-for-instant representation of building response during earthquake excitation is one way to infer damage patterns, correlate changes in frequency with changes in physical behavior of the building, etc.

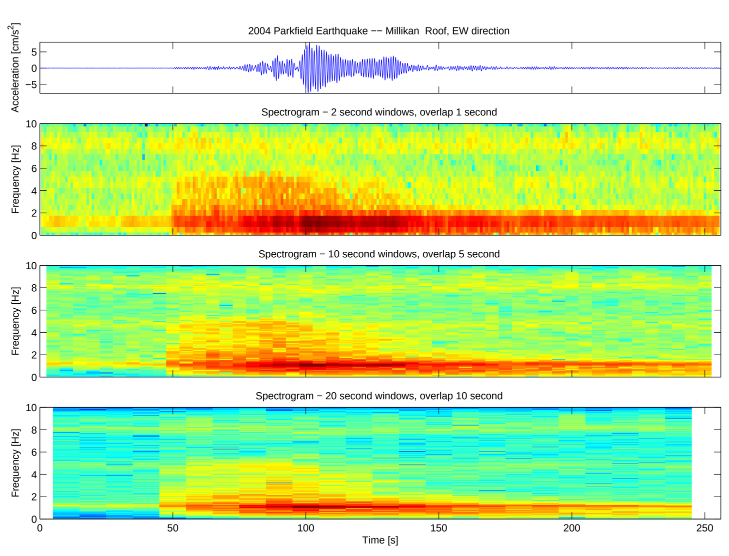

Figure 6: Parkfield Earthquake, Millikan Library. Spectrograms at different windowing resolutions, to illustrate the trade-off in temporal and frequency resolution.

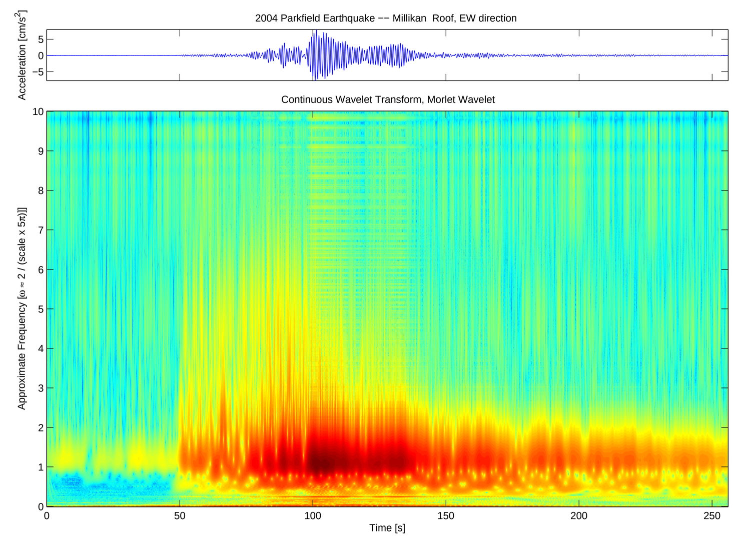

Figure 7: Continuous Wavelet Transformation. Parkfield Earthquake, Millikan Library. Excellent temporal resolution, but has poor frequency resolution, due to the scale-frequency conversion being only an approximation.

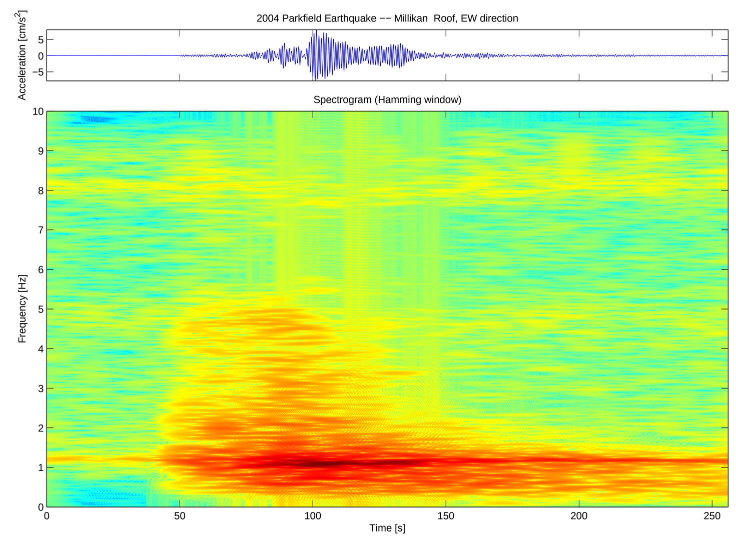

Figure 8: Spectrogram (Hamming window). Parkfield Earthquake, Millikan Library. Again, the spectrogram introduces energy in the time-frequency plane before the onset of the event.

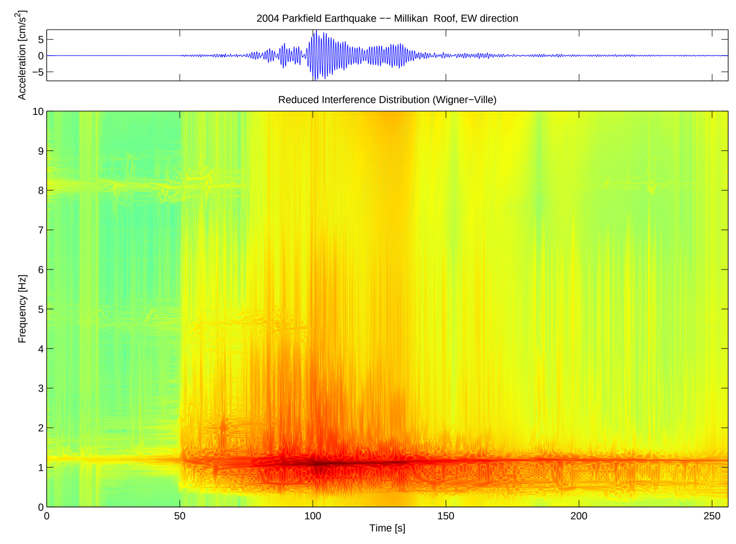

Figure 9: Reduced Interference Distribution (Wigner-Ville). Parkfield Earthquake, Millikan Library. This representation has many advantages over spectrogram and scalogram (wavelet) methods. The RID has very crisp temporal and frequency resolution, while the interference terms have been effectively reduced as compared with the Wigner-Ville distribution.

Conclusions

The Wigner-Ville Distribution is optimal in many ways for creating an instantaneous estimation of frequency content in a signal. It has a penalty of introducing interference terms, but this can be mitigated by using one of the many reduced interference distributions. With a goal of identifying the onset of changes in dynamic properties, we have developed a framework in which to apply modern time-frequency analysis techniques to data from instrumented structures.

With the increased number of instrumented civil engineering structures, there is a growing interest in real-time damage detection, which corresponds in practice to identifying changes in the dynamic properties of the structure. These variations in dynamic properties are connected to changes in global stiffness, which has important implications for structural health monitoring techniques.

References

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[2] P. Flandrin and W. Martin. *The Wigner Ville Spectrum of Nonstationary Random Signals*. "The Wigner Distribution: Theory and Applications in Signal Processing," edited by Mecklenbräuker, W. and Hlwatsch, F., ELSEVIER Ltd, 1997.

[3] J. Ville. Theorie et applications de la notion de signal analytique. *Cables et Transmissions*, 2A(1):61—74, 1948. I. Selin, translator, *"Theory and applications of the notion of complex signal,"* RAND Corporation Technical Report T-92, Santa Monica, CA (1958).

[4] E. Wigner. On the quantum correction for thermodynamic equilibrium. *Physical Review*, 40:749—759, June 1932.

For more on the time-frequency representations used here, please see:
http://ce.caltech.edu/case/tfr/