VARIATIONS IN THE DYNAMIC PROPERTIES OF STRUCTURES: THE WIGNER-VILLE DISTRIBUTION

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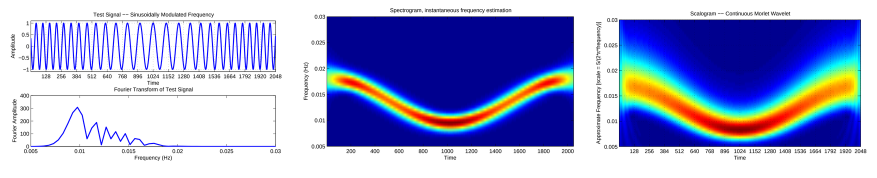
Introduction

The Wigner-Ville Distribution (WVD) is a promising method for analyzing frequency variations in seismic signals, including those of interest for structural monitoring. Nonlinearities in the force displacement relationship will temporarily decrease the apparent natural frequencies of structures during strong to moderate excitation, and earthquake damage can permanently change building stiffnesses. A Fourier Transform of a building record contains information regarding frequency content, but it can not resolve the exact onset of changes in natural frequency – all temporal resolution is contained in the phase of the transform. The spectrogram is better able to resolve temporal evolution of frequency content, but has a trade-off in time resolution versus frequency resolution in accordance with the uncertainty principle. To overcome this restriction, several distributions have been proposed, including wavelet methods. Wavelets have very good temporal resolution, but generate a wavelet "scale" instead of frequency, which complicates analysis of evolving frequency content. Time-frequency transformations such as the WVD allow for instantaneous frequency estimation at each data point, for a typical temporal resolution of fractions of a second. We develop a mathematical foundation for analyzing the evolution of frequency content in a signal, and apply these techniques to synthetic records from linear and nonlinear FEM analysis (including plastic rotation and weld fractures). Our analysis techniques are then applied to earthquake records from damaged buildings.

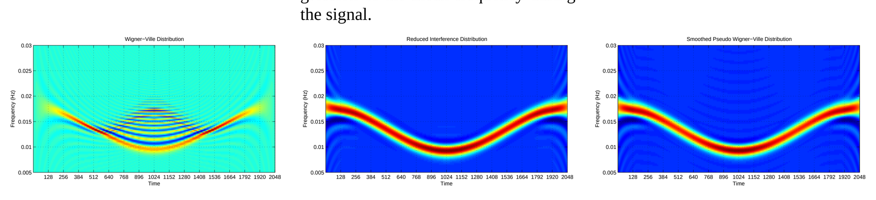
Time-Frequency Representations



(a) Sample signal with evolving frequency, and Amplitude of the Fourier Transform – Note that the Fourier Transform, while correctly identifying the components of the signal, does not allow for a straightforward interpretation of frequency evolution.

(b) Spectrogram, this distribution has limited resolution and takes on a 'blocky' structure due to the trade-off between temporal resolution and frequency resolution in the time-frequency plane – however, this representation clearly captures information about progression of the mean frequency during the signal.

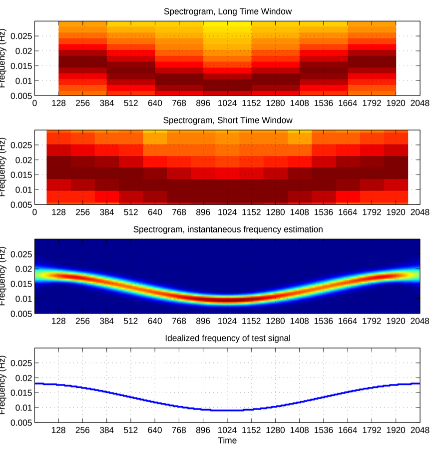
(c) Scalogram of sample signal – Continuous Morlet wavelet transformation. Wavelet transformations create a time-scale representation, where scale has an approximate equivalency to frequency.



(d) Wigner-Ville Distribution for the sample signal. Note that resolution of the signal is quite crisp, though the strong interference terms make this method unsuitable for general signal analysis.

(e) Reduced Interference Distribution for the sample signal. Adapted from the WVD, with a smoothing kernel to reduce interference terms.

(f) Smoothed Pseudo Wigner-Ville Distribution for the sample signal. An alternate method of reducing interference.



(g): Time-frequency representations of sample signal: Spectrogram (long time window), Spectrogram (short time window), Spectrogram (instantaneous frequency estimation, using a Hanning windowing technique), Theoretical frequency content of the signal. These plots demonstrate the typical trade-off in time resolution against frequency resolution – A longer Fourier Transform window improves the frequency resolution, but will also smear the signal along the time axis; a shorter window will improve temporal resolution, at the cost of frequency resolution.

Figure 1: (a) Sample signal , (b) Spectrogram, (c) Scalogram (Wavelets), (d) Wigner-Ville Distribution, (e) Reduced Interference Distribution, (f) Smoothed Pseudo Wigner-Ville Distribution, and (g) Spectrogram resolution example.

Wigner-Ville Distribution

For a signal, $s(t)$, with analytic associate $x(t)$, the Wigner-Ville Distribution, $WVD_x(t,\omega)$ is defined as:

$$WVD_x(t,\omega) = \int_{-\infty}^{+\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})\mathrm{e}^{-i\omega\tau}\,d\tau$$

This distribution was first introduced by E. Wigner in the context of quantum mechanics [6], and later independently developed by J. Ville who applied the same transformation to signal processing and spectral analysis [5].

- The WVD is similar to the Fourier Transform, though, instead of transforming the original signal, the kernel of the WVD contains a type of autocorrelation term (in this case, the phase lag of the ambiguity function, or "...properly symmetrized covariance function..." [3]).
- The analytic associate $x(t)$ of a signal $s(t)$ is here defined such that $x(t) \equiv s(t) + iH[s(t)]$, where $H[s(t)]$ is the Hilbert Transform of the signal $s(t)$.
- In this study, time-frequency analysis techniques are applied to the analytic associates of real signals unless noted otherwise – in particular, $x(t)$ is generally the complex-valued analytic associate of some real-valued time signal of interest.

In addition to being an entirely real-valued function, the WVD also satisfies the marginal and total energy conditions:

$$\int_{-\infty}^{+\infty} WVD_x(t,\omega)\,d\omega = |x(t)|^2$$

$$\int_{-\infty}^{+\infty} WVD_x(t,\omega)\,dt = |\mathbb{X}(\omega)|^2$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} WVD_x(t,\omega)\,dt\,d\omega = \mathbb{E}_x$$

...where $\mathbb{X}(\omega)$ is the Fourier Transform of $x(t)$, and $\mathbb{E}_x$ is the total energy of signal $x(t)$. The integral of the WVD along the time axis is equal to the power spectrum of the signal (the squared Fourier Transform), while the integral along the frequency axis gives the squared envelope of the original time series – the energy condition states that the total area of the WVD, the double integral across time and frequency, is the energy contained in the original signal. These conditions have an intuitive appeal, as they imply a limitation on the extent of the signal in the time-frequency plane. In an ideal representation, a signal of short duration would have a narrow representation along the time axis, and the frequency content would be localized to the frequency of the signal. Meeting the marginal conditions is one way in which optimal time-frequency representations can be constructed, though these conditions are neither necessary nor sufficient for the construction of useful representations.

Reduced Interference Distribution

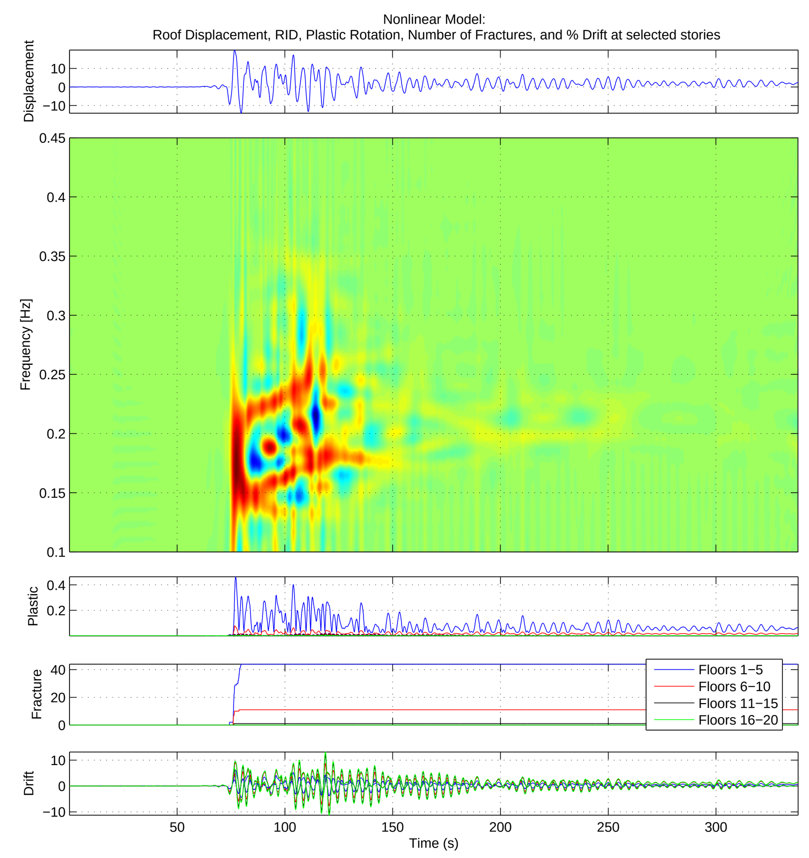
In general, a Reduced Interference Distribution (RID) refers to any distribution that reduces the expression of the cross-terms relative to the auto-terms in a quadratic time-frequency representation [1]. One such RID uses a smoothing kernel based on a Hanning window:

$$RID(t,\omega) = \int_{-\infty}^{+\infty} h(\tau)R_x(t,\tau)\mathrm{e}^{-i\omega\tau}\,d\tau$$

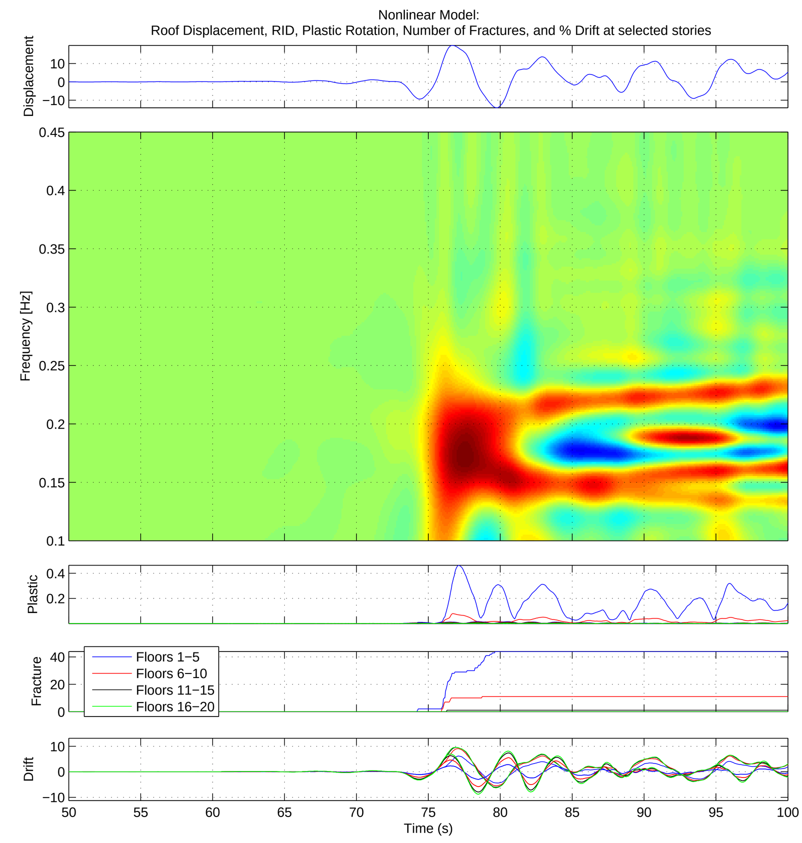
$$R_x(t,\tau) = \int_{-\frac{|\tau|}{2}}^{+\frac{|\tau|}{2}} \frac{g(\nu)}{|\tau|}(1+\cos\frac{2\pi\nu}{\tau})x(t+\nu+\frac{\tau}{2})x^*(t+\nu-\frac{\tau}{2})\,d\nu$$

...where $h(\tau)$ is a time smoothing window, and $g(\nu)$ is a frequency smoothing window. Depending on the implementation, the smoothing functions $h(\tau)$ and $g(\nu)$ can be adjusted to match the requirements of the data.

Damage Detection: Synthetic Building Records



(a)



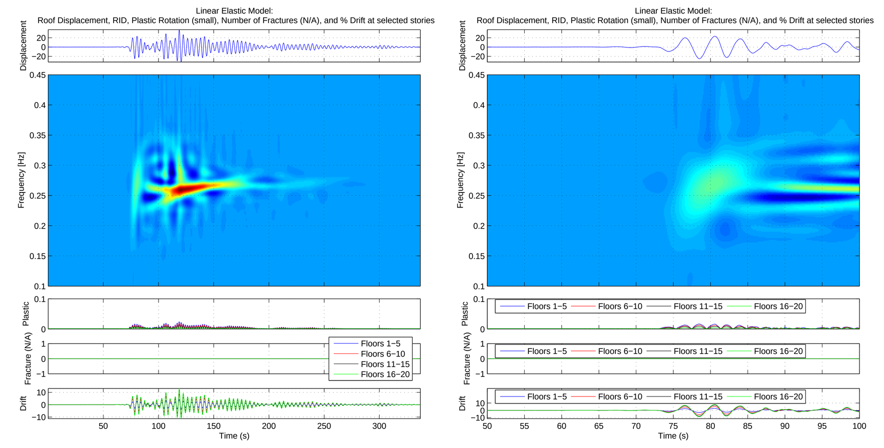
(b)

Figure 2: **(a)** RID of synthetic data for the nonlinear FEM model. (2003 Tokachi-Oki earthquake.) The top plot is the roof displacement, the large center plot is the RID of the data, and the three bottom plots present the plastic rotation, number of fractured welds, and drift for selected stories. The initial strong motion pulse is magnified in **(b)**, to more clearly show the evolution of frequencies on a time scale of seconds, and show the correlation with the selected damage measures.

With a goal of identifying the onset of changes in dynamic properties, we have developed a framework in which to apply modern time-frequency analysis techniques to data from civil structures under earthquake loading. The goal in these studies is to obtain a detailed, instant-for-instant representation of the dynamic properties of a structure, and use these changes in properties to infer damage patterns.

We have applied time-frequency analysis techniques to a finite-element model that includes varying sources of nonlinearity.

- Planar-frame fiber model includes material nonlinearity, geometric nonlinearity (P-Δ effects, member buckling), and weld fractures [4].
- 20-story steel moment-frame building, with height of 78.26m above ground, was designed according to UBC94, and input to the finite-element program.
- Linear and nonlinear models were subjected to the strong ground motions recorded at station HKD095 during the 2003 Tokachi-Oki earthquake (Mw 8.3).

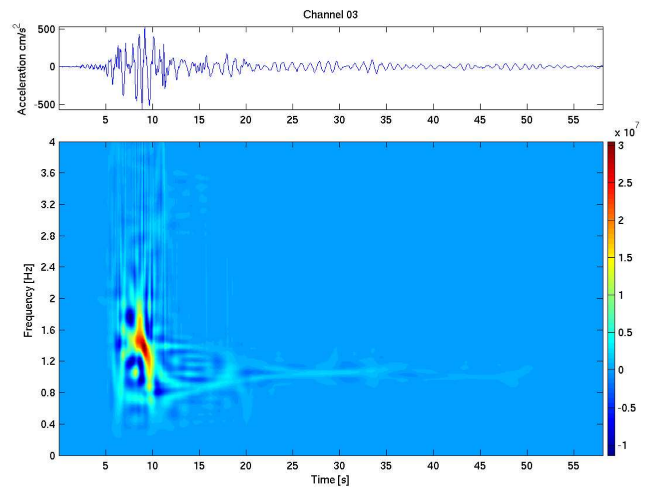


(a) (b)

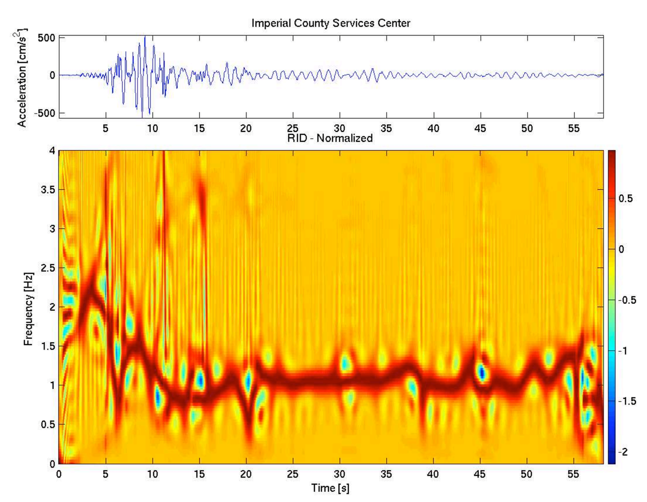
Figure 3: RID of synthetic data for the linear FEM model. (2003 Tokachi-Oki earthquake.) Model is not truly linear, as there is some plastic behavior of the joints. In this case, the building has no weld fracture, and has a much smaller change in dynamic properties as compared to the nonlinear case. Plots as in Figure 2.

Damage Detection: Instrumented Buildings

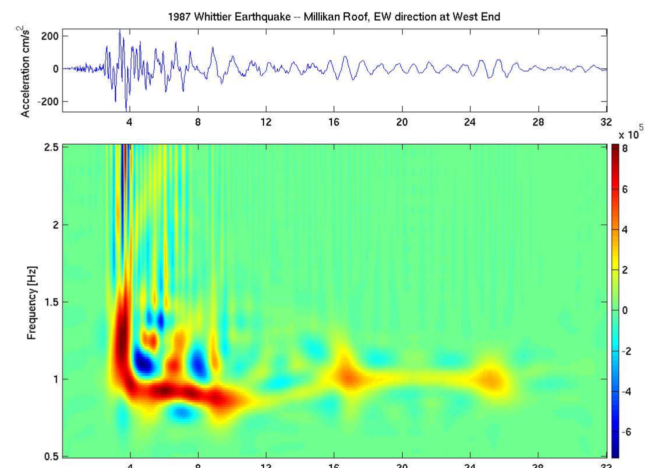
Changes in apparent instantaneous frequency during an event can be permanent (e.g. damage) or temporary (e.g. nonlinear elasticity). Care needs to be taken when using changes in dynamic properties to infer damage. The time-frequency analysis techniques described in this study were applied to instrumented buildings damaged in historical earthquakes; the Imperial County Services Building (El Centro) was famously damaged in the 1979 Imperial Valley event, and Caltech's Millikan Library had a permanent change in natural frequencies from the 1987 Whittier Narrows event.



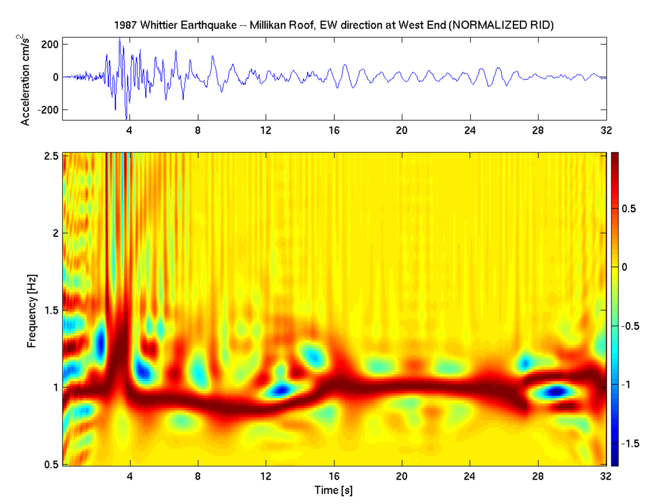
(a) RID of Imperial County Services Building, NS roof records for 1979 Imperial Valley event. This building had badly damaged columns and was later demolished.



(b) RID of Imperial County Services Building, scaled by peak amplitude at each time instant.



(c) RID of Millikan Library, EW 9th floor records for 1987 Whittier Narrows event. No obvious damage, pre-event frequency of 1.21Hz was permanently changed to 1.18Hz (-2.5%) by this event. The change in frequency is from unknown causes, but is believed to be from non-structural damage to the foundation-soil system.



(d) RID of Millikan Library, scaled by peak amplitude at each time instant.

Figure 4: Behavior of instrumented buildings during strong earthquakes. – Imperial County Services Building, El Centro, during 1979 Imperial Valley Event: a) RID b) Normalized RID – Millikan Library, during 1987 Whittier Narrows event: c) RID d) Normalized RID – Adapted from [2].

Conclusions

The Wigner-Ville Distribution, and related joint time-frequency representations, allow for an instantaneous estimation of frequency content in instrumented structures. This is a valuable tool for structural health monitoring, and for evaluating changes in dynamic properties that occur over fractions of a second. The WVD is an optimal time-frequency representation in many ways, with advantages over spectrogram and wavelet methods, but it has a penalty of introducing artifacts (which can be mitigated with the Reduced Interference Distribution).

Acknowledgements

The authors thank the CSMIP program for the Imperial County Services Center records, Professor John Hall (Caltech) for the nonlinear finite element program, Matt Muto (Caltech) for assistance with obtaining the Millikan Library / Whittier Narrows records, and Germán Prieto (UCSD) for his work applying joint time-frequency analysis methods to seismic records.

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For more on the time-frequency representations used here, please see:
http://ce.caltech.edu/case/tfr/